

# Chiral condensates and size of the sigma term

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## Abstract

The in-medium chiral condensate is studied with a new approach which has the advantage of no need for extra assumptions on the current mass derivatives of model parameters. It is shown that the pion-nucleon sigma term is  $9/2$  times the average current mass of light quarks, if quark confinement is linear. Considering both perturbative and non-perturbative interactions, the chiral condensate decreases monotonously with increasing densities, approaching to zero at about  $4 \text{ fm}^{-3}$ .

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The calculation of in-medium quark condensates is of crucial importance to the chiral property of quantum chromodynamics (QCD) [1]. A popular method for this is the Feynman-Hellmann theorem which gives [2]

$$\frac{\langle \bar{q}q \rangle_{n_b}}{\langle \bar{q}q \rangle_0} = 1 - \frac{1}{2|\bar{q}q|_0} \frac{\partial E}{\partial m_0}, \quad (1)$$

where  $E$  is the energy density above the vacuum,  $m_0$  is the current mass of  $u/d$  quarks, and  $|\bar{q}q|_0 \equiv |\langle \bar{q}q \rangle_0| = -\langle \bar{q}q \rangle_0 \approx (225 \text{ MeV})^3$ . Eq. (1) can be schematically derived as such. From Feynman-Hellmann theorem one has  $\langle \Psi(\lambda) | \frac{d}{d\lambda} H(\lambda) | \Psi(\lambda) \rangle = \frac{d}{d\lambda} \langle \Psi(\lambda) | H(\lambda) | \Psi(\lambda) \rangle$  for the hamiltonian  $H(\lambda)$  with the eigenstate  $|\Psi(\lambda)\rangle$ . Write the QCD hamiltonian density for the two flavor symmetric case as  $H_{\text{QCD}} = H' + 2m_0 \bar{q}q$  where the second term breaks the chiral symmetry explicitly while the first term is the remaining part which has nothing to do with the current mass  $m_0$ . Then making the substitution  $\lambda \rightarrow m_0$  and  $H \rightarrow \int d^3x H_{\text{QCD}}$  gives  $2\langle \Psi(m_0) | \bar{q}q | \Psi(m_0) \rangle = \frac{\partial}{\partial m_0} \langle \Psi(m_0) | H_{\text{QCD}} | \Psi(m_0) \rangle$ . Here the integration over space is canceled due to the uniformity of the system. The derivative has been changed to partial derivative because the system energy may depend on other independent quantities for example the density  $n_b$ . Apply the equality just obtained to the state  $|\Psi\rangle = |n_b\rangle$  and  $|\Psi\rangle = |0\rangle$ , and then taking the difference lead to Eq. (1) naturally.

Because no one can exactly solve QCD presently, the energy density  $E$  in Eq. (1) is given with some model parameters. The main difficulty in this formula is that one has to know the current mass derivatives of model parameters which are, except for a few special cases, usually not available.

Recently, another approach has been proposed [3,4] with the advantage of no need for assumptions on the current mass derivatives of model parameters. However, how the effective Fermi momentum is connected to density was not investigated clearly. In Ref. [3], it is merely boosted by a simple factor of 2 while not boosted in the section 2 of Ref. [4], compared with the non-interacting case. In this paper, it will be generally proved that the relation between the effective Fermi momentum and density is determined by general principles of thermodynamics. At lower densities, a simple relation among the pion-nucleon sigma term, the quark current mass, and quark confinement is found, which means that, if the confinement is linear, the pion-nucleon sigma term should be 9/2 times the average current mass of light quarks. Full density behavior of the in-medium chiral condensate is calculated by considering both perturbative and non-perturbative interaction effects, which gives a zero condensate at about  $4 \text{ fm}^{-3}$ .

Let's outline the key points of the new approach in Refs. [3,4] for the two flavor case. Ignoring terms breaking flavor symmetry, the corresponding QCD hamiltonian density can be schematically written as  $H_{\text{QCD}} = H_k + 2m_0 \bar{q}q + H_I$ ,

where  $H_k$  is the kinetic term,  $H_I$  is the interacting part, and  $m_0$  is the average current mass of  $u$  and  $d$  quarks. Define a new operator  $H_{\text{eqv}} \equiv H_k + 2m\bar{q}q$  with  $m$  being an equivalent mass to be determined by the requirement that it meets  $\langle H_{\text{eqv}} \rangle_{n_b} - \langle H_{\text{eqv}} \rangle_0 = \langle H_{\text{QCD}} \rangle_{n_b} - \langle H_{\text{QCD}} \rangle_0$ .

Since particles considered are uniformly distributed, or in other words,  $n_b$  has nothing to do with space coordinates, one can write  $\langle \Psi | m(n_b) \bar{q}q | \Psi \rangle = m(n_b) \langle \Psi | \bar{q}q | \Psi \rangle$ . This equality is especially obvious if it is considered in terms of quantum mechanics:  $|\Psi\rangle$  is a wave function with arguments  $n_b$  and coordinates, the expectation value is nothing but an integration with respect to the coordinates. Therefore, if  $n_b$  does not depend on coordinates, the function  $m(n_b)$  is also a coordinate-independent c-number, and can naturally be taken out of the integration. However, if  $n_b$  is local, the case becomes much more complicated and it will not be considered here. Now substituting the expressions of  $H_{\text{QCD}}$  and  $H_{\text{eqv}}$  into the equation  $\langle H_{\text{eqv}} \rangle_{n_b} - \langle H_{\text{eqv}} \rangle_0 = \langle H_{\text{QCD}} \rangle_{n_b} - \langle H_{\text{QCD}} \rangle_0$  leads to

$$m = m_0 + \frac{\langle H_I \rangle_{n_b} - \langle H_I \rangle_0}{2(\langle \bar{q}q \rangle_{n_b} - \langle \bar{q}q \rangle_0)} \equiv m_0 + m_I. \quad (2)$$

Therefore, considering quarks as a free system, i.e., without interactions, while keeping the system energy unchanged, quarks should acquire an equivalent mass of the form shown in Eq. (2). From this equation one can see that the equivalent mass  $m$  includes two parts: one is the original mass or current mass  $m_0$ , the other one is the interacting part  $m_I$ . Obviously the equivalent mass is a function of both the quark current mass and the density. At finite temperature, it depends also on the temperature as well. Because the hamiltonian density  $H_{\text{eqv}}$  has the same form as that of a system of free particles with a density dependent mass  $m$ , the corresponding dispersion relation  $\varepsilon = \sqrt{p^2 + m^2}$  is also density dependent. The energy density of quark system can then be expressed as

$$E = \frac{g}{2\pi^2} \int_0^{p_f} \sqrt{p^2 + m^2} p^2 dp = \frac{gp_f^3}{6\pi^2} m F\left(\frac{p_f}{m}\right), \quad (3)$$

where  $g = 12$  is the degeneracy factor, and the function  $F$  is defined to be  $F(x) \equiv \frac{3}{8}[x\sqrt{x^2 + 1}(2x^2 + 1) - \text{sh}^{-1}(x)]/x^3$  with  $\text{sh}^{-1}(x) = \ln(x + \sqrt{x^2 + 1})$ . The effective Fermi momentum  $p_f$  is related to the chemical potential  $\mu$  by

$$p_f = \sqrt{\mu^2 - m^2} \text{ or } \mu = \sqrt{p_f^2 + m^2}. \quad (4)$$

The physical meaning of the equivalent mass is that quarks should have the equivalent mass if the system is free but with unchanged energy. It is not

difficult to prove that such an equivalent mass always exists. In principle, if one obtains the energy density  $E$  from some realistic models or even from QCD in the future, the equivalent mass  $m$  can be obtained by solving Eq. (3). From the point of view of thermodynamics, one can always choose to express some quantity as an ideal gas form with density and/or temperature dependent particle mass(es) while all other quantities determined by thermodynamic principles. For example, in Refs. [5,6], the pressure of gluon plasma is expressed as the same form with that of an ideal gas while the gluon ‘thermo mass’  $m(T)$  is determined by fitting to lattice data. In Ref. [7], the entropy is expressed as the same form as that of an ideal gas. Here the energy has been expressed as the same form as that of an ideal quark gas. However, how the Fermi momentum  $p_f$  is connected to density should be studied carefully. If no interactions are accounted or a specific duality principle is assumed, the Fermi momentum will be connected to density simply by  $p_f = [(18/g)\pi^2 n_b]^{1/3}$ . Here quark interactions are included within the equivalent mass. So its relation to density should be determined by general thermodynamic principles. It can be shown that the baryon number density  $n_b$  is related to the Fermi momentum  $p_f$  by

$$n_b = \frac{gp_f^3}{18\pi^2} + \frac{g}{12\pi^2} \int \left[ p_f - \frac{m^2 \text{sh}^{-1}(p_f/m)}{\sqrt{p_f^2 + m^2}} \right] m dm. \quad (5)$$

To prove this, let’s write the fundamental thermodynamic relation  $d(V E) = Td(V S) - PdV + 3\mu d(V n_b)$  which is the combination of the first and second laws of thermodynamics. Here  $P$  is the pressure,  $S$  is the entropy density, and  $V$  is the volume. Now rewrite the equality as  $dE = TdS - (P + E - TS - 3\mu n_b)dV/V + 3\mu dn_b$  which implies  $P + E - TS - 3\mu n_b = 0$  and  $\partial E/\partial n_b = 3\mu$ . At zero temperature, the entropy is zero (the third law of thermodynamics), these two equalities become

$$P = -E + 3\mu n_b, \quad dn_b = dE/(3\mu). \quad (6)$$

In the present approach, the energy density is given in Eq. (3) from which one has  $dE = \frac{\partial E}{\partial p_f} dp_f + \frac{\partial E}{\partial m} dm$ . Substituting this into the second equality of Eq. (6) and then integrating over both sides will give Eq. (5) naturally.

Defining  $E_I \equiv \langle H_I \rangle_{n_b} - \langle H_I \rangle_0$ , the interacting part of the equivalent mass,  $m_I$  in Eq. (2), can be re-written as  $m_I = E_I/(3n^*)/(1 - \langle \bar{q}q \rangle_{n_b}/\langle \bar{q}q \rangle_0)$  with  $n^* \equiv (2/3)|\bar{q}q|_0$ . Solving for the ratio  $\langle \bar{q}q \rangle_{n_b}/\langle \bar{q}q \rangle_0$ , this equation leads to

$$\frac{\langle \bar{q}q \rangle_{n_b}}{\langle \bar{q}q \rangle_0} = 1 - \frac{E_I}{3n^* m_I}. \quad (7)$$

According to the expression of  $H_{\text{QCD}}$ , the total energy density of the quark

system can be expressed as

$$E = \frac{g}{2\pi^2} \int_0^{p_{f0}} \sqrt{p^2 + m_0^2} p^2 dp + E_I \equiv E_0 + E_I. \quad (8)$$

The first term is the energy density without interactions, the second term  $E_I$  is the contribution from interactions, and the Fermi momentum  $p_{f0}$  for the non-interacting case is connected to density by  $p_{f0} = [(18\pi^2/g)n_b]^{1/3}$ .

On the other hand,  $E$  has already been expressed in Eq. (3). So, replacing the  $E$  on the left hand side of Eq. (8) with the right hand side of Eq. (3), then dividing by  $3n_b$ , one has

$$\left(\frac{p_f}{p_{f0}}\right)^3 m F\left(\frac{p_f}{m}\right) - m_0 F\left(\frac{p_{f0}}{m_0}\right) = \frac{E_I}{3n_b}. \quad (9)$$

Substituting Eq. (9) into Eq. (7) leads to

$$\frac{\langle \bar{q}q \rangle_{n_b}}{\langle \bar{q}q \rangle_0} = 1 - \frac{n_b}{n^* m_I} \left[ \frac{p_f^3}{p_{f0}^3} m F\left(\frac{p_f}{m}\right) - m_0 F\left(\frac{p_{f0}}{m_0}\right) \right]. \quad (10)$$

At the same time, taking the derivative with respect to  $n_b$  on both sides of Eq. (3) gives

$$\sqrt{p_f^2 + m^2} = \frac{1}{3} \frac{dE}{dn_b}. \quad (11)$$

Therefore, if one knows the energy density  $E$ , the equivalent mass  $m$  and effective Fermi momentum  $p_f$  can be obtained by solving the Eqs. (11) and (3), and the chiral condensate can then be calculated from Eq. (10). The great advantage of this scheme to calculate the in-medium chiral condensate is that one does not need to make any assumption on the current mass derivatives of model parameters. In fact, the current mass dependence can be derived like this. Substituting Eq. (3) into Eq. (1), performing the derivative with respect to  $m$  at fixed  $p_f$ , then comparing with Eq. (10), one will have

$$\frac{\partial m}{\partial m_0} = \frac{m F(p_f/m) - (p_{f0}/p_f)^3 m_0 F(p_{f0}/m_0)}{m_I f(p_f/m)} \quad (12)$$

with  $f(x) \equiv \frac{3}{2}[x\sqrt{x^2+1} - \ln(x + \sqrt{x^2+1})]/x^3$ .

Let's compare the present equation of state to the famous MIT bag model. According to Eqs. (6), (3), and (5), the pressure of a cold quark plasma is

$$P = \frac{gm^4}{48\pi^2} \left[ \frac{\mu}{m} \sqrt{\frac{\mu^2}{m^2} - 1} \left( 2\frac{\mu^2}{m^2} - 5 \right) + 3\text{ch}^{-1} \left( \frac{\mu}{m} \right) \right] + \frac{g\mu}{4\pi^2} \int \left[ \sqrt{\mu^2 - m^2} - \frac{m^2}{\mu} \text{ch}^{-1} \left( \frac{\mu}{m} \right) \right] m dm, \quad (13)$$

where  $\text{ch}^{-1}(x) \equiv \ln(x + \sqrt{x^2 - 1})$ . For comparison purpose, the independent state variable has been changed to the chemical potential  $\mu$  through Eq. (4). Obviously, the second term is the correspondence of the bag constant  $B$ . The difference is that the bag constant here is not really a constant. Instead, it is density-dependent.

Because baryon masses depend on the in-medium chiral condensate, the total energy of the system also depends on it [8]. In principle, the hadronic matter below chiral phase transition point, where the quark condensate is nonzero, should be described in terms of hadronic degree of freedom. Otherwise, both perturbative and nonperturbative quark interactions, including quark confinement, should be accounted [9]. So let's expand the equivalent mass to Laurent series ( $m_I$  must be divergent at  $p_f = 0$  or  $n_b = 0$  due to quark confinement), and merely take the leading terms in both directions:

$$m_I = \frac{a_{-1}}{p_f} + a_1 p_f. \quad (14)$$

In the following, it will be seen that the first term is non-perturbative, mainly originated from the linear confinement of quarks, and the second term is from the leading contribution of perturbative interactions with the coefficient  $a_1$  connected to the QCD coupling  $\alpha_s$  by

$$a_1 = \sqrt{2\alpha_s/(\pi - 2\alpha_s)}. \quad (15)$$

It is not possible to compare the present formalism at finite density with lattice results. However, there are several expressions for the pressure of a cold quark plasma, e.g., those from the hard-thermal-loop perturbation theory [10] and from the weak-coupling expansion [11,12]. Although they are different in higher orders, their leading term is identical. Let's assume the interacting equivalent mass  $m_I$  is, at the perturbative densities and to leading order, also proportional to the chemical potential  $\mu$ , i.e.,  $m_I = \alpha_0 \mu$ . It is shown in the following that the coefficient is  $\alpha_0 = \sqrt{2\alpha_s/\pi}$ . In fact, the pressure in Eq. (13) can be expanded to Taylor series at  $m_I = 0$ , i.e.,

$$\begin{aligned}
P = P_{\text{id}} - \frac{gm_0}{4\pi^2} & \left\{ \left[ \mu \sqrt{\mu^2 - m_0^2} - m_0^2 \text{ch}^{-1} \left( \frac{\mu}{m_0} \right) \right] m_{\text{I}} \right. \\
& - \mu \int \left[ \sqrt{\mu^2 - m_0^2} - \frac{m_0^2}{\mu} \text{ch}^{-1} \left( \frac{\mu}{m_0} \right) \right] dm_{\text{I}} \Big\} \\
& - \frac{g}{8\pi^2} \left\{ \left[ \mu \sqrt{\mu^2 - m_0^2} - 3m_0^2 \text{ch}^{-1} \left( \frac{\mu}{m_0} \right) \right] m_{\text{I}}^2 \right. \\
& - \mu \int \left[ \sqrt{\mu^2 - m_0^2} - 3\frac{m_0^2}{\mu} \text{ch}^{-1} \left( \frac{\mu}{m_0} \right) \right] dm_{\text{I}}^2 \Big\} \\
& + \text{higher order terms}
\end{aligned} \tag{16}$$

where  $P_{\text{id}} \equiv g/(48\pi^2)[\mu\sqrt{\mu^2 - m_0^2} (2\mu^2 - 5m_0^2) + 3m_0^4 \text{ch}^{-1}(\mu/m_0)]$  is the pressure of a degenerate non-interacting quark plasma. The convergence of Eq. (16) can be mathematically proven. Now substituting  $m_{\text{I}} = \alpha_0 \mu = \sqrt{2\alpha_s/\pi} \mu$  into the above perturbative expansion, and then taking the limit of  $m_0 \rightarrow 0$  due to the extreme smallness of the current mass of light quarks, the second term vanishes while the first and third terms give  $P/P_{\text{id}} = 1 - 2\alpha_s/\pi$  which is consistent with the hard-thermal-loop resummed pressure [10] and the weak-coupling expansion [11,12].

At the same time, the second term of Eq. (14) dominates at higher densities, i.e.,  $m_{\text{I}} = a_1 p_{\text{f}}$ . So inserting  $p_{\text{f}} = m_{\text{I}}/a_1$  into Eq. (4) and solving for  $m_{\text{I}}$  give  $m_{\text{I}} = a_1 [\sqrt{(1 + a_1^2)\mu^2 - m_0^2} - a_1 m_0]/(1 + a_1^2)$  which approaches, at the limit of  $m_0 \rightarrow 0$ , to  $m_{\text{I}} = a_1 \mu / \sqrt{1 + a_1^2}$ . Comparing this with  $m_{\text{I}} = \alpha_0 \mu$  gives  $a_1 / \sqrt{1 + a_1^2} = \alpha_0$ . Solving for  $a_1$  from this equality then leads to Eq. (15).

In order to perform the integration in Eq. (5), one should know the total derivative of Eq. (14), i.e.,

$$\frac{dm_{\text{I}}}{dp_{\text{f}}} = \frac{\partial m_{\text{I}}}{\partial p_{\text{f}}} + \frac{\partial m_{\text{I}}}{\partial a_1} \frac{da_1}{dp_{\text{f}}} = -\frac{a_{-1}}{p_{\text{f}}^2} + a_1 + p_{\text{f}} \frac{da_1}{dp_{\text{f}}}. \tag{17}$$

From Eqs. (4), (15), and the Gell-Mann-Low equation  $\mu d\alpha_s/d\mu = \sum_{n=0}^{\infty} C_n \alpha_s^{n+2}$  [13], it can be shown that the derivative of  $a_1$  with respect to  $p_{\text{f}}$  is

$$\frac{da_1}{dp_{\text{f}}} = \frac{p_{\text{f}} + m \partial m / \partial p_{\text{f}}}{\sum_{n=0}^{\infty} C_n \alpha_s^{n+2} (p_{\text{f}}^2 + m^2) - m \frac{\partial m}{\partial a_1}} \tag{18}$$

with  $\partial m / \partial p_{\text{f}} = -a_{-1}/p_{\text{f}}^2 + a_1$ ,  $\partial m / \partial a_1 = p_{\text{f}}$ ,  $d\alpha_s/da_1 = \pi a_1 / (1 + a_1^2)^2$ . To second order, the Gell-Mann-Low equation can be integrated out, giving

$$\frac{1}{\alpha_s} - \frac{1}{\alpha_s(1)} + \frac{C_1}{C_0} \ln \frac{C_1 + C_0/\alpha_s(1)}{C_1 + C_0/\alpha_s} + C_0 \ln \frac{\mu}{\Lambda} = 0, \tag{19}$$

where  $\alpha_s(1)$  is the value of  $\alpha_s$  at the QCD scale point  $\mu = \Lambda$ ,  $C_0 = -(11N_c - 2N_f)/(6\pi)$  and  $C_1 = -[(34N_c - 13N_f)N_c + 3N_f/N_c]/(24\pi^2)$  are the Gell-Mann-Low coefficients [14]. The scale parameter  $\Lambda$  is usually taken to be 300 MeV while  $\alpha_s(1)$  is taken to be 1 [11]. The confinement parameter  $a_{-1}$  should satisfy the constraint: the energy per baryon for the two-flavor case is no less than 930 MeV, in order not to contradict standard nuclear physics [15]. In the present calculation,  $a_{-1}$  is taken to be such a value that the maximum value of the QCD running coupling  $\alpha_s$  just does not exceed 1 at whole densities. If larger  $a_{-1}$  values are used, the condensate goes to zero more rapidly.

For a given  $p_f$ , the running coupling  $\alpha_s$  is obtained by solving Eq. (19) with the help of Eqs. (4), (14), and (15). Inserting Eq. (17) with Eq. (18) into Eq. (5) will give the baryon number density  $n_b$ . The chiral condensate can then be calculated from Eq. (10).

Now let's find the lower density behavior of Eq. (10). Because  $m \approx m_I \gg p_f$  and  $F(p_f/m) \approx 1$  at lower densities, Eqs. (11) and (9) become  $m_I = dE_I/(3dn_b)$  and  $(p_f/p_0)^3 m_I = E_I/(3n_b)$ . Denoting the inter-quark interaction by  $v(\bar{r})$  with  $\bar{r} \propto 1/n_b^{1/3}$  being the average distance of quarks [3]. The interacting energy per baryon  $E_I/(3n_b)$  is proportional to  $v(\bar{r})$ . Because the confinement interaction dominates at lower densities while the confinement is linear according to lattice simulations [16], one has  $E_I/(3n_b) \propto \bar{r}^Z$ . Here the confinement exponent is denoted by  $Z$ . For linear confinement, it is equal to unity. With a proportion coefficient  $\sigma$ , one can write  $E_I = 3\sigma n_b^{1-Z/3}$ . Substituting this into the approximate equalities of Eqs. (11) and (9) just obtained gives  $m_I = \sigma(1 - Z/3)/n_b^{Z/3}$  and  $p_f = [(18\pi^2/g)n_b/(1 - Z/3)]^{1/3}$ . These two equalities give an account for the validity of the first term in Eq. (14). They also show that the effective Fermi momentum has been boosted to a higher value by a factor, i.e.,  $p_f^3/p_0^3 = 1/(1 - Z/3)$ . This ratio can also be obtained by expanding the integrand of the integration in Eq. (5) to Taylor series at  $p_f = 0$ , taking the leading term  $2p_f^3/(3m)$ , making the variable substitution  $m_I = \alpha_{-1}/p_f^Z$  [ $Z=1$  corresponds to the first term of Eq. (14)], and then completing the integration. Finally substituting the ratio into Eq. (10) gives  $\langle \bar{q}q \rangle_{n_b}/\langle \bar{q}q \rangle_0 = 1 - n_b/n_l$  with  $n_l = (1 - Z/3)n^*$ .

Now demanding a compatibility with the famous model-independent result in nuclear matter, i.e.,  $\langle \bar{q}q \rangle_\rho/\langle \bar{q}q \rangle_0 = 1 - \rho/\rho^*$  with  $\rho^* \equiv M_\pi^2 F_\pi^2/\sigma_N$  [2], one can get, from  $n_l = \rho^*$ , an interesting relation

$$\sigma_N = \frac{9m_0}{3 - Z} \quad (20)$$

which relates the pion-nucleon sigma term  $\sigma_N$ , the confinement exponent  $Z$ , and the average current mass of  $u/d$  quarks whose value is about  $m_0 = 7.5$  MeV. In obtaining the above equation, the Gell-Mann-Oakes-Renner relation



$-2m_0\langle\bar{q}q\rangle_0 = M_\pi^2 F_\pi^2$  has been applied. For the linear confinement of  $Z = 1$ , Eq. (20) gives  $\sigma_N = \frac{9}{2}m_0 \approx 34$  MeV. The reported  $\sigma_N$  values in literature are significantly different. Previously, it was reported to be a smaller one, e.g., in the range of 25–26 MeV [17]. Later, a bigger value, e.g.,  $\sigma_N \approx 56.9 \pm 6.0$  MeV [18] was arrived at. After that, modest values of about 45 MeV was obtained [19,20]. The accepted value used to be as high as around 65 MeV [21]. Recently, a different value of about 50 MeV was reported again [22]. This phenomena is due to the fact that  $\sigma_N$  is not directly measurable but extrapolated according to special models. Obviously, the  $\sigma_N$  value here is bigger than previously determined, but smaller than presently accepted. Naturally, if one would like to reproduce the bigger values, the confinement would have to be also bigger. However, lattice simulations favor the linear confinement [16].

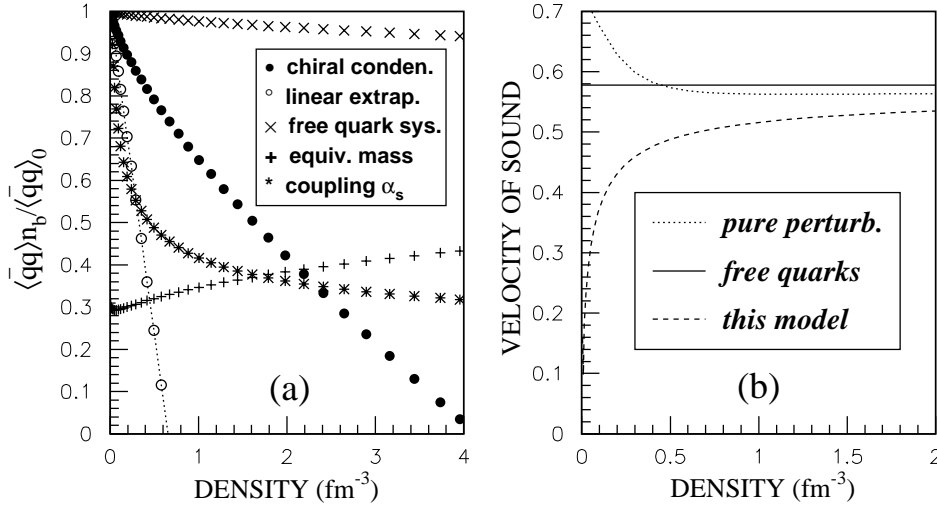


Fig. 1. (a) Density dependence of the in-medium chiral condensate. It is generally a decreasing function of the density (star line), approaching to zero at about  $4 \text{ fm}^{-3}$ . The open-circle line on the left is the linear extrapolation. The up-most line with crosses is for a quark system without interactions. The plus-marked line gives the corresponding equivalent mass, in unit of the nucleon mass 939 MeV. The star line is the QCD coupling  $\alpha_s$ . (b) Velocity of sound in the pure perturbative calculation (the dotted line) and in the present approach (the dashed line).

Numerical results are plotted in Fig. 1. In Fig. 1 (a), the up-most line is for the chiral condensate in a free quark gas. Because the decreasing speed is proportional to the current mass which is very small, the line goes down slowly. When interactions are considered, the condensate (full circles) decreases much more rapidly, approaching to zero at about  $4 \text{ fm}^{-3}$  which is still an extremely high density. The linear behavior is shown with an open-circle line on the left. The equivalent mass is given, in unit of the nucleon mass  $M_N = 939$  MeV, with a plus-marked line, which shows that the ratio  $m/M_N$  is about 0.3–0.4. In constituent quark models [23], the constituent quark mass is usually taken to be  $M_N/3$ , consistent with the present result.

Equations (13) and (3) provide the equation of state. If the energy per baryon  $E/n_b$  is plotted as a function of the density  $n_b$ , the minimum point should correspond exactly to the zero pressure, which is a general requirement of thermodynamics, as has been shown in Ref. [24]. However, this is not the case in the pure perturbative results, e.g., a check of the figure 7 in Ref. [11] shows that the density corresponding to zero pressure is smaller than the density corresponding to the minimum energy per quark. This has a strong consequence on the velocity of sound, calculated by  $|dP/dE|^{1/2}$ , which has been shown in Fig. 1 (b) for the present approach (the dash line) and in the weak expansion (the dotted line). At higher densities, they approach asymptotically to the ultra-relativistic case (the full line) as expected. However, their lower density behavior is completely opposite. This may have some stringent consequences on dynamic situations, such as the possibility of strange candidates in neutron stars or perhaps heavy ion collisions.

In summary, the density behavior of the in-medium chiral condensate has been studied with a new approach. The chiral condensate is shown to be generally a decreasing function of density. At lower densities, it decreases linearly, which means that the pion-nucleon sigma term is 9/2 times the average current mass of light quarks if the confinement is linear. With increasing densities, the deviation from linear extrapolation becomes significant. Considering both perturbative and non-perturbative effects, the condensate approaches to zero at about  $4 \text{ fm}^{-3}$ .

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